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Calculations of Temperature, Conductive Heat Flux, and Heat Wave Velocities Due to Radiant Heating of Opaque Materials

by Arthur Cohen

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14. ABSTRACT The analytic solutions of the one-dimensional Fourier conductive heat flux law and corresponding transient heat transfer equation have been used to calculate temperature, conductive heat flux, and their trajectories due to radiant heating of opaque materials. Heat wave trajectories (and velocities) are defined by the value of a constant temperature or constant conductive heat flux that propagates through the materials. For constant values of zero, the analytic solutions lead to infinite heat wave velocities. For nonzero constant values, the numerical solutions at ultra-short transient times result in other nonphysical velocities (i.e., >3E(10) cm/s). These calculations demonstrate that there are limitations to the validity of the Fourier heat flux law and corresponding heat transfer equation, although they have been used successfully to solve engineering and scientific heat conduction problems for over 180 years.						
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1. Introduction

The objective of this report is to document recent one-dimensional (1-D) calculations of temperature, conductive heat flux, and heat wave velocities for an opaque, semi-infinite cylinder due to radiant heating. The Fourier conductive heat flux law is assumed to be valid. An idealized model is used for the calculations. Radiant flux enters the planar surface. It is uniform and steady with beam and cylinder diameters equal. The initial temperature is uniform, and thermal properties are constant. Heat wave trajectories (and velocities) are defined by the value of a constant temperature or constant conductive heat flux that propagates through the opaque material. For the constant values of zero, the analytic solutions (*1*) of the 1-D Fourier conductive heat flux law and transient heat transfer equation (derived by applying energy conservation to the Fourier heat flux law) lead to infinite heat wave velocities. For the constant values >0, the numerical solutions at ultra-short transient times result in other nonphysical velocities (i.e., >3E(10) cm/s).

These calculations suggest that there are limitations to the validity of the Fourier heat flux law and heat transfer equation, although they have been used successfully to solve engineering and scientific heat conduction problems for over 180 years.

2. Nomenclature

Table 1 shows the units and values of the thermophysical properties and F that were used in all calculations. The thermophysical values chosen were those for RDX at T ~ 288 K (*2*).

Table 1. Nomenclature.

Symbols	Quantity	Units	Values
t	radiant flux time	s	—
d	distance from the planar surface	cm	—
V	temperature increment, i.e., V (d,0) = 0	K	—
f	conductive heat flux	cal/cm^2-s	—
λ	thermal conductivity	cal/cm-s-K	2.5×10^{-4}
ρ	density	g/cm^3	1.76
c	heat capacity	cal/g-K	0.3
α	thermal diffusivity = $\lambda/\rho \cdot c$	cm^2/s	4.7×10^{-4}
F	radiant flux	cal/cm^2-s	100

3. Mathematical Formulation

For the simplified model with opaque (surface absorption) materials, the Fourier heat flux law is

$$f(d,t) = -(1/\lambda)\cdot\frac{\partial}{\partial d}V(d,t) \tag{1}$$

with boundary condition

$$f(d,t) = F; \quad d = 0, t > 0 \ . \tag{2}$$

The Fourier heat transfer equation is

$$\frac{\partial}{\partial t}V(d,t) = \alpha\frac{\partial^2}{\partial d^2}V(d,t) \tag{3}$$

with boundary condition

$$\frac{\partial}{\partial d}V(d,t) = -F/\lambda \ ; \ d = 0, t > 0 \, . \tag{4}$$

Using equations 1 and 3, Carslaw and Jaeger (*1*) show that the Fourier flux transfer equation is

$$\frac{\partial}{\partial t}f(d,t) = \alpha\frac{\partial^2}{\partial d^2}f(d,t) \ ; \ d > 0 \, , t > 0 \tag{5}$$

with the boundary condition

$$f(d,t) = F \quad ; \quad d = 0 \, . \tag{6}$$

The solution for f(d,t) (*1*) is

$$f(d,t) = F\cdot\operatorname{erfc}\left(\frac{d}{2\cdot\sqrt{\alpha\cdot t}}\right). \tag{7}$$

The solution for V(d,t), obtained by integration of equation 1, is (*1*)

$$V(d,t) = 2\cdot\frac{F}{\lambda}\left(\sqrt{\frac{\alpha\cdot t}{\pi}}\ \exp\left(\frac{-d^2}{4\alpha t}\right) - \frac{d}{2}\cdot\operatorname{erfc}\left(\frac{d}{2\cdot\sqrt{\alpha\cdot t}}\right)\right). \tag{8}$$

Figures 1 and 2 are calculated values of f (cal/cm^2-s) and V (K) vs. d (cm) at t (s) = 0.001, 0.01, 0.05, and 0.1.

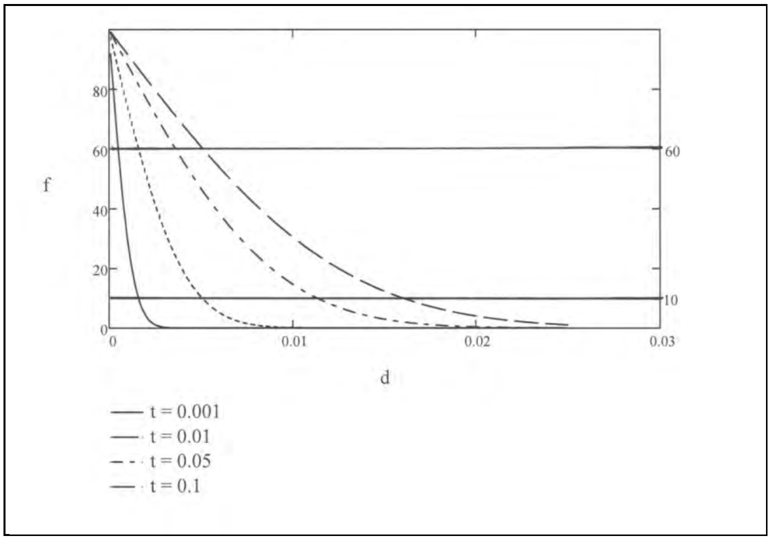

Figure 1. Conductive flux vs. distance.

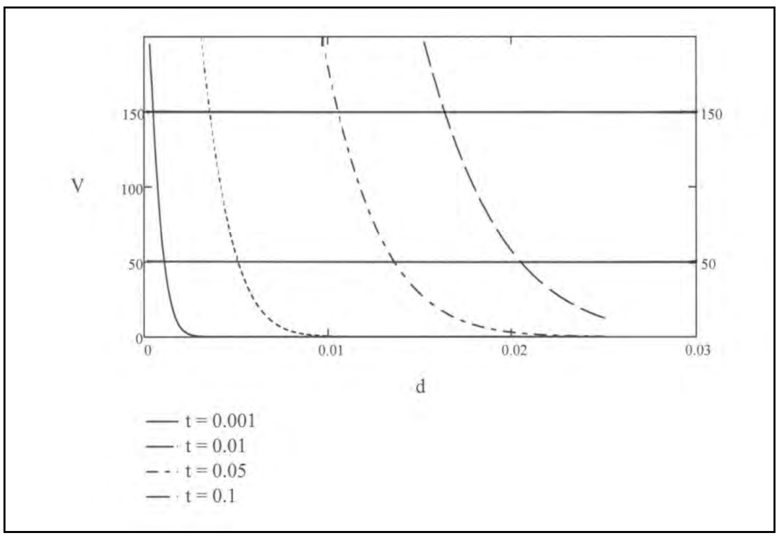

Figure 2. Temperature vs. distance.

Average heat wave velocities for conductive flux at f = 10 and 60 and for temperature at V = 50 and 150 can be obtained from the distances between intersections of their corresponding lines with those calculated for different values of t. Distances vary inversely with the values of both f(d) and V(d), which suggests that the largest heat wave velocities are for f = 0 and V = 0. To obtain heat wave velocities at points on trajectories, let x = trajectory distance (cm) from the planar surface.

Values of x at given values of t can be obtained from numerical solutions of equations 7 and 8. For f = 0 and V = 0, the numerical solutions (*3*) are unstable. To calculate these trajectories,

let $z = \dfrac{x}{2\cdot\sqrt{\alpha\cdot t}}$ and substitute into equations 7 and 8.

$$f\,(z) = F\cdot \mathrm{erfc}(z)\,. \tag{7a}$$

$$V(z,t) = 2\cdot\frac{F}{\lambda}\cdot\sqrt{\frac{\alpha\cdot t}{\pi}}\cdot e^{z^2}\cdot\left(1-\sqrt{\pi}\cdot z\cdot e^{z^2}\cdot \mathrm{erfc}(z)\right). \tag{8a}$$

The trajectories $(x = 2\cdot z\cdot\sqrt{\alpha\cdot t})$ can be calculated for given values of t from solutions of equations 7a and 8a for z.

When f(z) = 0, then erfc(z) = 0. The solution to equation 7a is z = ∞.

When V(z,t) = 0, then

$$\sqrt{\pi}z\cdot e^{z^2}\cdot \mathrm{erfc}(z) = 1\,. \tag{9}$$

Using L'Hopital's rule, when z = ∞, then

$$\sqrt{\pi}z\cdot e^{z^2}\cdot \mathrm{erfc}(z) = 1\,. \tag{10}$$

The solution to equation 8a is z = ∞.

For V > 0 and f > 0, the trajectories $(x = 2\cdot z\cdot\sqrt{\alpha\cdot t})$ can be calculated from the values of x corresponding to given values of t from numerical solutions of equations 7 and 8 or the corresponding z from equations 7a and 8a.

Corresponding velocities $\frac{d}{dt}x$ (fv for flux and Vv for temperature) can be calculated by numerical differentiation. Calculating trajectories with z = ∞ is a problem. The heat wave velocities can also be obtained analytically from the calculated trajectories using g(x,t) = constant ; $\frac{d}{dt}x = -\left(\frac{\partial}{\partial t}g(x,t)\cdot\frac{1}{\frac{\partial}{\partial x}g(x,t)}\right)$.

When f(x,t) = g(x,t), then

$$fv = \frac{x}{2\cdot t} = z\cdot\sqrt{\frac{\alpha}{t}}\,. \tag{11}$$

When f(x,t) = 0, then z = ∞.

When x = 0, then z < ∞ , i.e., $0 < x \le \infty$.

When x = ∞ and t = ∞, then z = 1 < ∞, i.e. , $0 \le t < \infty$.

Using equation 11 with $0 < x \le \infty$ and $0 \le t < \infty$,

$$fv = \infty. \tag{11a}$$

When V(x,t) = g(x,t), then

$$Vv = \frac{e^{-\frac{x^2}{4 \cdot t \cdot \alpha}} \sqrt{\frac{\alpha}{t}}}{\sqrt{\pi} \cdot \mathrm{erfc}\left(\frac{x}{2 \cdot \sqrt{t \cdot \alpha}}\right)} = \frac{z\sqrt{\frac{\alpha}{t}}}{\sqrt{\pi} \cdot z \cdot e^{z^2} \cdot \mathrm{erfc}(z)}. \tag{12}$$

When V(x,t) = 0, then $z = \infty$. Using equation 10, $\sqrt{\pi} z \cdot e^{z^2} \cdot \mathrm{erfc}(z) = 1$.

$$Vv = z \cdot \sqrt{\frac{\alpha}{t}}. \tag{13}$$

When $0 < x \le \infty$ and $0 \le t < \infty$, then

$$Vv = \infty, \tag{13a}$$

Equations 11a and 13a show that when $0 < x \le \infty$ and $0 \le t < \infty$, and $f(z) = V(z,t) = 0$, then $Vv = fv = \infty$. This is consistent with the results shown in figures 1 and 2.

Figures 3 and 4 show the calculated values of x(cm) and vV(cm/s) vs. t(s) for constant values of V. The x ratios (RxV) and vf ratios (RvV) are shown on the right y-axis. In figure 4, the vertical line indicates that for V = 1E(–19) ; vV > 3E10 at $t < 1.37 \cdot 10^{-23}$.

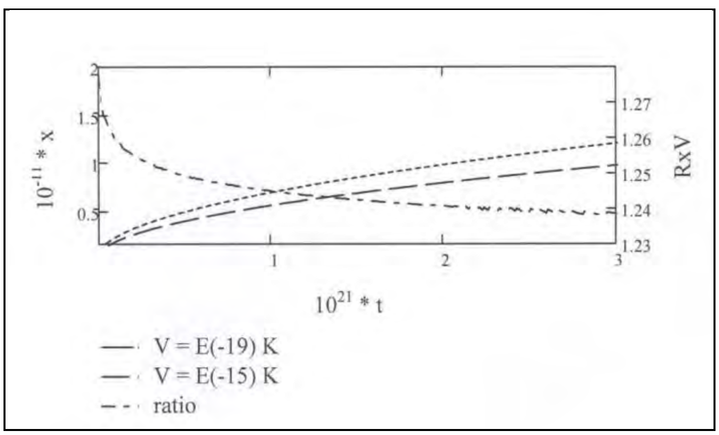

Figure 3. Temperature trajectory distance vs. time.

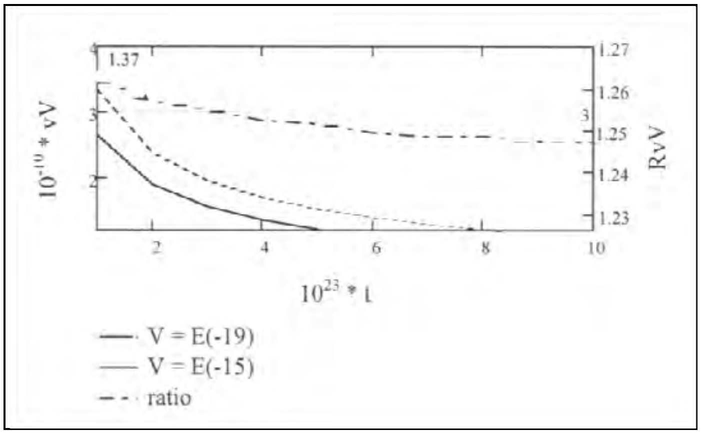

Figure 4. Temperature wave velocity vs. time.

Figures 5 and 6 correspond to figures 3 and 4 for the calculated values of x and vf vs. time for constant values of f/F. The x ratios (Rxf) and vf ratios (Rvf) are shown on the right y-axis. In figure 6, the vertical lines indicate that for f/F = 1E(–10) and 1E(–16), vf > 3E(10) at t < $1.1 \cdot 10^{-23}$ and $1.8 \cdot 10^{-23}$, respectively.

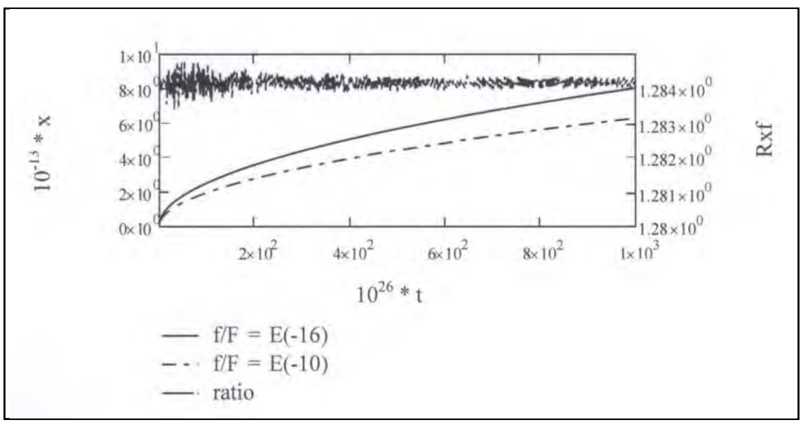

Figure 5. Flux wave trajectory distance vs. time.

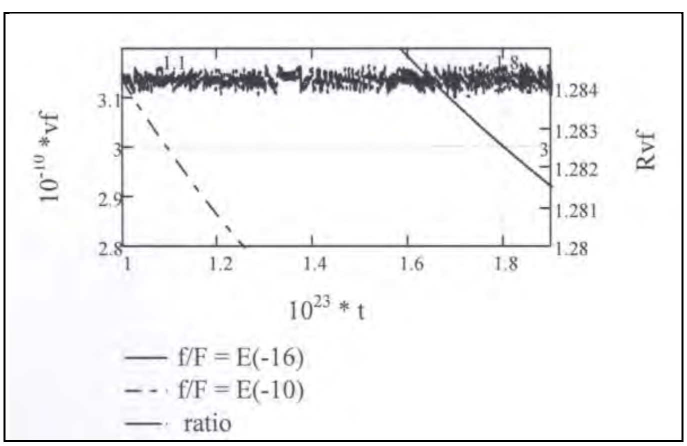

Figure 6. Flux wave velocity vs. time.

4. Conclusion

The calculations of nonphysical heat wave velocities for temperature and heat flux using analytic solutions of the Fourier heat flux law and the corresponding transient heat transfer equation indicate that there are limitations to their validity. This problem has been well noted in the heat transfer literature (*4*). The inability of the Fourier heat transfer (parabolic) equation to predict measurements of heat propagation velocities in solids near zero absolute temperatures and for pulsed laser processing of solid surfaces involving ultra-short transient heating times is probably due to the Fourier flux law being constitutive and not valid under these extreme conditions. The Fourier equations are based on the linear dependence of heat flux and temperature gradient. The details of the mechanism for thermal energy transfer (molecular collisions in the absence of net mass motion) are still under investigation. Consideration of phonon interactions as a molecular gas (*5*) has recently been used to describe the thermal energy transfer mechanism for conductive heat flux.

Non-Fourier-type conductive heat flux laws in solids have been proposed and include both time and temperature gradient dependence (*4*). These laws combined with energy conservation can lead to hyperbolic type transient heat transfer equations, which avoid the Fourier problem of nonphysical heat propagation velocity predictions and give better agreement with measurements under extreme conditions. Under normal conditions, these time-dependent conductive heat flux laws transit into the Fourier temperature gradient flux law.

Temperature measurements (using different techniques) of the rate at which equilibrium is established when solids (meat) at different (near room) temperatures are placed in contact have been used to compare heat transfer velocities with predictions of the Fourier and non-Fourier type conductive heat transfer equations in solids (*6*). The conclusions as to which type equation is in better agreement with the data are in disagreement. It is of interest to determine the lower time limits at which Fourier's heat transfer equation remains valid. It may be possible to obtain estimates from the published results of recent pulsed radiant heating experiments on opaque and semi-transparent solids.

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